

## Applications of Differentiation

2. a) Area of garden =  $400 \text{ m}^2$

Length of garden =  $x \text{ m}$

length =  $L$

Area = length  $\times$  width

$$L \times x + x = 400$$

$$L \times x = 400 + x$$

$$L = \frac{400}{x} + 1$$

b)  $\frac{\partial L}{\partial x} = \frac{d}{dx} \left( x + \frac{400}{x} \right)$

$$= \frac{d}{dx} (x) + 400 \frac{d}{dx} \left( \frac{1}{x} \right)$$

$$L = 1 - \frac{400}{x^2}$$

stationary points of  $L$

$$1 - \frac{400}{x^2} = 0$$

$$\frac{400}{x^2} = 1$$

$$x^2 = 400$$

$$x = 20$$

$$c) \quad x = 20.$$

$$h = 20$$

Minimum point (0,0)

Maximum point (20,40)

3. a) ~~Area~~

$$\text{Volume} = \pi r^2 h = \frac{1}{2} \text{ litre}$$

$$\pi r^2 h = 500 \text{ ml}$$

$$h = \frac{500}{\pi r^2}$$

b) Surface area =  $2\pi rh + \pi r^2$

$$\pi r^2 h = \text{Volume} = 1000$$

$$S.A = \frac{\pi r^2 h}{r} + \pi r^2$$

$$A = \pi r^2 + \frac{1000}{r}$$

$$g) = \frac{1000}{r} + \pi r^2$$

$$f'(r) = \frac{d}{dr} 1000r^{-1} + \pi r^2$$

$$f'(r) = -1000r^{-2} + \pi r$$

$$f''(r) = (-1)(-2)1000r^{-3} + \pi$$

$$= 2(1000r^{-3}) + \pi$$

$$= \frac{20000}{r^3} + \pi = 0$$

$$\frac{20000}{r^3} = -\pi$$

$$r^3 = \frac{20000}{-\pi}$$

$$r = \left( \frac{20000}{-\pi} \right)^{\frac{1}{3}}$$

$$r = 18.53$$